Disturbance Propagation in Power System Based on an Epidemic Model

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***Abstract*—Study the dynamic behavior of disturbance propagation is of great importance for the safety and stability of power system. The dynamic analysis of the disturbance of the power system attempts to use an mature epidemic model of biology and open a new path to the dynamics of disturbance propagation. In this paper, based on the analogy epidemic model and the propagation of the grid disturbance, the concept of disturbance propagation intensity was put forward. Furthermore, the complex network theory was used to analyze the complex network topology of the power system, and the random matrix theory was used to analyze the relationship between the spatial and temporal data in power system. At last, the dynamic model of disturbance propagation in power system based on data-driven was established. The simulation shows that the proposed model can effectively supplement the traditional power system disturbance propagation analysis, and can analyze the dynamic behavior of power system.**

Keywords—epidemic model; data analysis; complex network; random matrix theory; disturbance propagation

I. INTRODUCTION

In recent years, with the rapid growth of power grid load, large-scale access to electric vehicles, grid connection of scale renewable energy power system, power market reform and other factors, the randomness of the power system gradually becomes an important reason for the safe and stable operation of the power system. The reduction of the safety and stability margin in power system makes the possibility of the whole system accident caused by the local disturbance greatly increased. The disturbance of the power system is unavoidable. If the disturbance effect is not obvious, through the dynamic analysis of disturbance propagation in power system, and then take the corresponding control measures, can reduce the impact of the disturbance [1-4]. Therefore, it is very important to study the dynamic characteristics of propagation on complex networks of power system. A good propagation dynamics model can be more accurate and intuitive understanding of the behavior of the network disturbance, and grasp the overall situation of the grid. In addition, it helps to identify the weak links of propagation behavior, accurately predict the possible degree of harm, and develop relevant control strategies.

At present, most of the propagation disturbance of power system is focused on the mechanism. James S. Thorp [5] presented the electromechanical wave propagation characteristics, equilibrium solutions, and linear stability. They also presented numerical simulations of the usual discrete generator model, based upon the swing equation, and demonstrated the electromechanical wave propagation as having interesting properties. Reference [6] discussed a disturbance produced in one point propagates to remote points of the system, it may be reflected at the boundaries and standing waves may build up. To obtain better insight into such phenomena, the complex power system is represented as a continuum, with transmission capacity and machine inertial uniformly distributed. In [7], a trajectory real-time prediction scheme based on the WAMS information is proposed for the multi-machine system. It can be seen that the dynamic theory of the traditional power system is based on the deterministic model. It didn’t consider the impact of random factors on the dynamic characteristics of the power system, there are obvious limitations [3]. With the rapid development of big data technology, study dynamic of power system disturbance based on data analysis is an important and urgent task.

We study the epidemic model in biology before studying the dynamic model of disturbance in power system. Epidemic model in addition to the medical industry, in the food industry, population dynamics and other progressive applications [8-10] also achieved some success. This paper is used in the power industry. The power system network belongs to the complex network and the propagation dynamics on the complex network is an important direction of the complex network research. It mainly studies the propagation mechanism and dynamic behavior of various complex networks in society and nature, and the control method which is feasible for these behaviors [11].

In this paper, complex network theory and the random matrix theory are used to build the model. Based on the topology of the power system, the dynamic behavior of power system disturbance is studied from the perspective of data driving which is an effective complement to the traditional power system disturbance propagation analysis. The relative importance index of each node is established by complex network topology analysis and the spatial and temporal data correlation for determining the propagation characteristics is analyzed by random matrix theory in the power system. On this basis, the disturbance propagation dynamic model is

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established by analyzing the propagation intensity, and the population behavior of the power system is analyzed and predicted. The validity and correctness of the proposed model are verified by comparing with the results of the simulation.

II. Dynamic Model of Disturbance Propagation

The disturbance propagation of complex power system can be regarded as obeying some kind of regular network propagation behavior. How to describe this kind of propagation behavior, reveal its characteristics, and find out the effective control method of the behavior, has always been the focus of attention. In fact, when the power system is affected by some kind of disturbance, the disturbance will find the weak part of the power system at first and then have an impact on it. After these success actions, it will not only have an impact on nodes, but also continue to spread out to influence more vulnerability systems.

The whole process of propagation is similar to the spread of the virus in the medical profession, as shown in Fig. 1. Biology has long begun to study the spread of the virus and established a relatively complete mathematical model of epidemic spread [11]. In [12], the SIR model of the epidemic model was used to evaluate the propagation of key nodes in the Power Grid of the western United States. Therefore, we can transplant some models used to study the spread of the virus to study the spread of disturbances in power system.

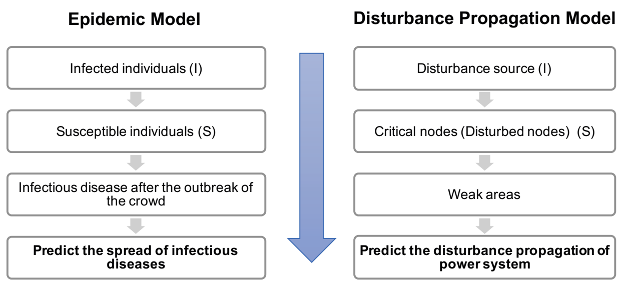

Fig. 1. Transmission of infectious disease and disturbance propagation of power system

A. Introduction to the Epidemic Model

The important basis of establishing the model of disturbance propagation model is the basic idea of the epidemic model. Based on the spread of infectious diseases, scientists involved a variety of network transmission models. At present, the most thorough research and the most widely used epidemic model are SI model, SIS model, SIR model and SIRS model. In the SI model, the network randomly selected one or a number of nodes for the infected node at the initial state, the rest are healthy nodes. The model uses the ratio of infection rate and cure rate as the infection intensity to measure the characteristics of a disease. At each time point, the infection rate is that if a healthy node is adjacent to one or more infected nodes, it becomes a disease node according to a predetermined probability. The cure rate means that each infected node becomes a healthy node according to a predetermined probability at the same time. These evolutionary rules are executed in parallel across the network. The greater the infection rate, the smaller the cure rate, the more likely the disease is to infect more people [11].

The traditional theory holds that a large propagation is possible only if the effective propagation rate exceeds a positive threshold. In many real networks, even if the effective propagation rate is very low, it may flood the entire network when the network exists infected nodes, such as the full outbreak of computer viruses. In the study of the disturbance dynamics in power system, a similar situation may occur when a certain disturbance occurs, such as a chain failure. Therefore, this paper is based on the SI model of infectious diseases to study the dynamics of disturbance in power system, which is the simplest propagation model in the epidemic model. In the SI model, the individual of the nodes is divided into S and I states.

S (Susceptible)——Susceptible state. Such individuals are generally healthy individuals, but can be infected by the virus.

I (Infected)——Infection status. Such individuals have been infected with the virus and have the ability to infect other healthy individuals.

At the beginning of the virus, one or several individuals in the network were infected with the virus and transmitted the virus to its neighbors by a certain probability. Once the number of susceptible individuals (often denoted by *S*) are infected, they become infected individuals (denoted *I*). And these individuals have become a new source of infection at this time, which can infect other individuals in the system. Similar to the chain failure in power system, we often use SI model for those who have infected the disease but can't cure, such as AIDS. The model has the following assumptions.

1) All individuals are vulnerable, that is, the virus is not immune, are likely to be infected.2) If an individual is infected, the individual will be in an infected state.3) Assume that the total number of individuals in the system is constant. In the initial epidemic model, the impact on the number of newborns, deaths, and migrants is negligible relative to the total number of systems in the short term. It is reasonable to assume that the total number of people in the system is constant.

Let $s(t)$ and $i(t)$ respectively denote the density of the network (the proportion of the total individuals' number) in the S-state and the I-state at time t. λ is the probability that S-state individuals are infected as I-state individuals, and N is the total number of individuals. Each infected individual will cause $\lambda s(t)$ individuals to be infected. The number of individuals infected is $Ni(t)$ in the network. So, the change rate of infected individual density over time is

$$\frac{dNi(t)}{dt} = Ni(t)\cdot \lambda s(t) \Rightarrow \frac{di(t)}{dt} = \lambda s(t)i(t) \tag{1}$$

All individuals in the system only have two states, *S* and *I*. So the number of individuals without infection with the rate of change is

$$\frac{ds(t)}{dt} = -\lambda i(t)s(t) \tag{2}$$

In the SI model, the dynamics of the virus infection can be described by the following differential equations:

$$\begin{cases} \frac{ds(t)}{dt} = -\lambda i(t)s(t) \\ \frac{di(t)}{dt} = \lambda i(t)s(t) \end{cases} \tag{3}$$

B. The Dynamic Model of Disturbance Propagation in Power System

When we built the dynamic model of disturbance propagation, we analogized the SI model and divided the various nodes into *S* and *I* two states in power system. When system disturbed, *S* represents a potentially disturbing node, which is a healthy node, but may be affected by disturbance propagation. *I* represents a node that has been affected by a disturbance, it has the ability to propagate to other unaffected health nodes. The related parameters of the epidemic model and the disturbance dynamics model are shown in Table 1.

TABLE I. ANALYTICAL MODEL PARAMETERS IN EPIDEMIOLOGY VERSUS POWER SYSTEM

Parameters	Epidemic Model	Disturbance Propagation Model
S	Susceptible state	Vulnerable nodes
I	Infection status	Disturbed nodes
N	The total population	The total nodes
$s(t)$	The individual density of S state at time t	The density of health nodes at time t
$i(t)$	The individual density of I state at time t	The density of disturbed nodes at time t
λ	The intensity of disease Infection	The intensity of disturbance propagation

In the transmission of the system disturbance, we believe that all nodes are fragile, likely to be disturbed. If a node is disturbed, the node is in a disturbed state and the total number of nodes in the system is constant. Therefore, the dynamic behavior of disturbance propagation can also be described by the differential equation (3).

Since all individuals in the system have only two states, i.e.

$$i(t) + s(t) = 1 \tag{4}$$

Assuming the initial density of the infected individual is $i(0)=i_0$ at the initial time. Equation (3) can be transformed into the following differential equation to solve the problem.

$$\begin{cases} \frac{di(t)}{dt} = \lambda i(t)s(t) = \lambda i(t)(1-i(t)) \\ i(0) = i_0 \end{cases} \tag{5}$$

Solve the differential equation, and we can get

$$i(t) = \frac{1}{1 + (1/i_0 - 1)e^{-\lambda t}} \tag{6}$$

In the SI model described above, it is mentioned that the intensity of infectious diseases transmission is determined by the infection rate and the cure rate. The main infection for the spread of the virus is contact, individual physical strength is different, the probability of infection is also different. The nature of the infectious disease itself and the topology of the network together determine the infectious disease transmission and process. For the disturbance propagation in power system, due to different types of disturbances, the state of each node in the system is different, the probability of being disturbed and the situation of further propagation are not the same. Therefore, the propagation intensity of each node is different.

How to determine the propagation intensity in the dynamic model of power system is worth studying. The characteristics of each node, the type of disturbance and the network topology together determine the dynamics of the system after the disturbance. The modeling of dynamic disturbance propagation model in power system is shown in Fig. 2.

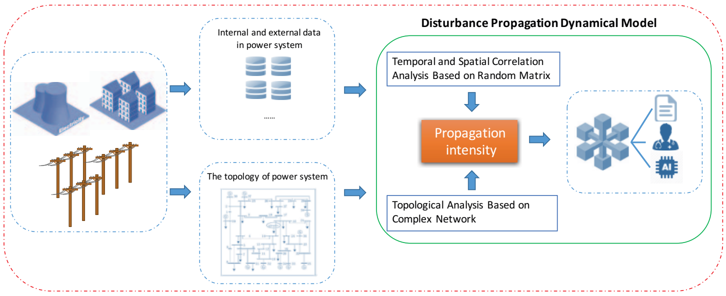

Fig. 2. Modeling of Disturbance Propagation Dynamical Model

III. PROPAGATION INTENSITY

Through the above analysis, the propagation intensity of the power system is determined by the topology and disturbance characteristics of the power system, i.e.

$$\lambda = f(K, \gamma, \tau) \tag{7}$$

Where λ is the propagation intensity, γ is the relative importance index of the neighboring nodes at the disturbance, which embodies the structural properties of nodes and is obtained by analyzing the topology of the power network based on complex network theory. τ is the statistical index of the spatial and temporal dynamic data of the power system, which changes with the time, and embodies the functional properties of nodes. It is obtained by the correlation analysis of the power system based on the random matrix theory. *K* is the correction coefficient. According to the method proposed in [13], all the node voltage of the system is followed by the adjoint matrix of each node. The eigenvalue statistics are obtained in succession, and the rate of change of the eigenvalue statistics at each time is obtained. Then set the threshold, so that it can correct by actually comparing the density of the disturbed nodes. *K* is related to fault type and threshold setting. As τ is a dynamic index, so the propagation intensity is a dynamic index. We can use the dynamic evolution process to analyze the relationship between λ and time point *t*, and obtain the dynamic behavior prediction of disturbance in power system.

A. Topological Analysis Based on Complex Network

The power system belongs to the complex network, which has the characteristic that the complex network has [11,14,15]. The size of the network is large, the number of network nodes can have thousands or even more, which makes large-scale network behavior with statistical characteristics. The connection structure is complex, and the network connection structure is neither completely regular nor completely random. But it has its inherent self-organization regularity, the network structure can present a variety of different characteristics. Node complexity is expressed as node dynamics complexity and diversity. The spatio-temporal network evolution process is complex, and power system has the complex evolution of time and space, which shows complex dynamic behavior. The network connection is sparse. The number of network connections with an N-node with a globally coupled structure is $O(N^2)$, and the actual number of large network connections usually is $O(N)$. The interaction between the multiple complexities mentioned above will lead to more unpredictable results.

The topology of the power system satisfies the complexity of the complex network mentioned above. For power system, the importance of each node is different in a complex network, so the same type of disturbance that occurs at different locations can have different effects on the propagation of the system. Therefore, we need to use quantitative analysis method at first to analyze the importance of nodes in the network, or analyze the importance of a node relative to one or more of the other nodes. In this paper, we use the subgraph index based on topology node correlation.

This method focuses on the direct connection between nodes and analyzes the function of the nodes by analyzing the topology. In a non-directional network, the importance of a node is represented by the number of connections between the target node and other neighboring nodes, that is, the nodes with the largest number of connections in the network play a key role in the network. This method can be described as, calculating the number of closed loops from one node to the end of the node, and a closed loop represents a subgraph in the network. The differences between nodes are presented by calculating the number of nodes participating in different subgraph and setting different weights for subgraphs.

The formula for calculating subgraph centrality of node v_i is

$$C_s(v_i) = \sum_{n=0}^{\infty} \frac{\mu_n(v_i)}{n!} \tag{8}$$

where $\mu_n(v_i) = (A^n)_{ii}$, $(A^n)_{ii}$ is the i[th] diagonal element of the n[th] power of the complex network adjacency matrix A. $\mu_n(v_i)$ is the number of loops which starting at node v_i via n consecutive edges back to node v_i. The relative importance of each node can be obtained by formula. After normalizing the index, the relative importance coefficient γ of the node is obtained.

B. Temporal and Spatial Correlation Analysis Based on Random Matrix

Power system is a complex network and has large scale, the number of nodes is thousands or even more. So the large-scale network behavior has statistical characteristics. The measurement data of the power system is based on GPRS, with accurate time stamps, so that all monitoring data in the same time profile, with space-time characteristics [16-17]. For internal and external data in the power system, multiple parameters are heterogeneous and difficult to find a mechanism or rule to accurately describe. Random matrix is a high-dimensional statistical analysis method for big data and random matrix theory provides a mathematical support for solving the statistical problems of these high-dimensional spaces, providing a powerful analytical approach to pure data-driven modeling [18-20]. This paper analyzes the spatio-temporal relation of disturbance in power system based on the random matrix theory.

According to the method mentioned in [13], the high dimensional random matrix ($X \in C^{N\times T}$) of the power grid is constructed by the data source *Ω*. Where *X* is the space-time section chosen according to the actual research problem, selects *N* from the n dimension in space and selects *T* from the *t* time period. The randomness of *X* is also reflected in the system spatio-temporal uncertainly data itself. The system is dynamically analyzed by sliding window. Select the current point in time t_0, the sliding time window width is T_w. Analysis matrix width is $t_0 — T_w$ to t_0 moment, that is, real-time dynamic analysis. This method can effectively remove the random noise generated during the acquisition and transmission from data.

The random matrix model constructed by a certain spatio-temporal section is $X_{N\times T_w}$, which is the data source for data analysis. Since these data are sampled in chronological order and have different spatial characteristics, the spatial and temporal characteristics of data are formed. We get the following formula at t_i.

$$X_{N\times T_w}(t_i) = \left[x(t_{i-T_w+1}), x(t_{i-T_w+2}), \quad , x(t_i)\right] \tag{9}$$

After normalizing the matrix, the sample covariance matrix is obtained by data transformation.

$$S_N = \frac{1}{N}\tilde{X}\tilde{X}' \tag{10}$$

Calculate the eigenvalues λ_{S_N} of the matrix S_N. In actual projects, the data often don't satisfy the independent distribution. However, through a variety of simulation and real system equipment testing, found that the results meet the M-P law [18,20,21]. Since the eigenvalues of the matrix are random, the linear eigenvalue statistics (LES) of the matrix are introduced. It is through the continuous test function φ to define:

$$\mu = \sum_{i=1}^{N} \varphi\left(\lambda_{S_N}\right) \tag{11}$$

The detection function is the core of the LES and requires sufficient continuity. In this paper, we use the Chebyshev polynomial.

$$\varphi(\lambda) = 2x^2 - 1 \tag{12}$$

The expected result and the expected value $E(\tau)$ ratio τ as a statistical indicator.

IV. Analysis Steps of Disturbance Propagation Dynamical Model

The key to the disturbance propagation dynamic model is to determine the propagation intensity. The model established in this paper not only considers the topology of a complex power network, but also considers the correlation between internal and external data of power system. The detailed analysis flow chart is shown in Fig. 3.

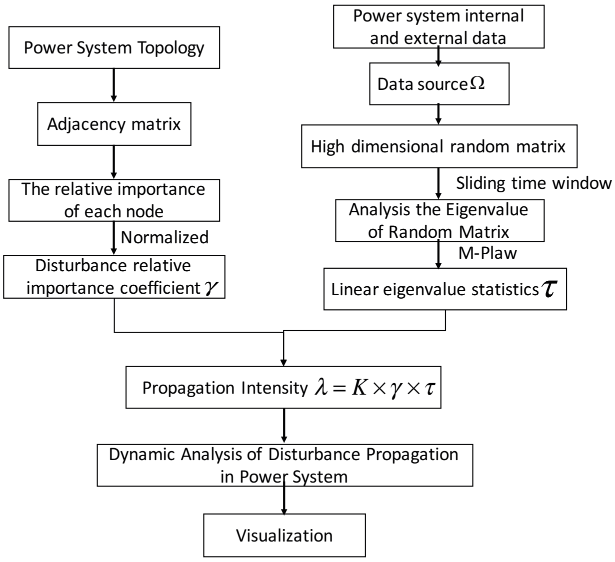

Fig. 3. Analysis steps of disturbance propagation in power system

V. Case Study

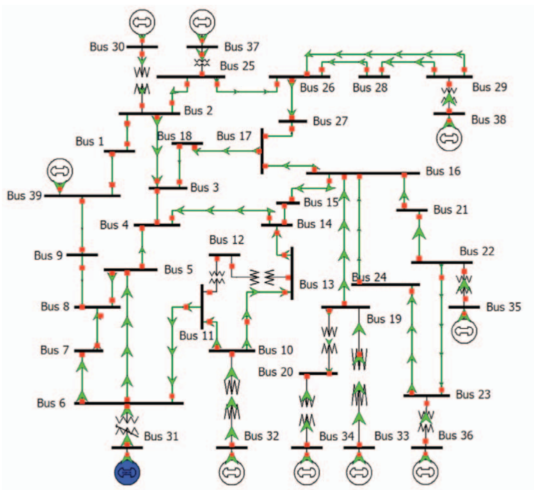

Fig. 4. Network of the IEEE 39-bus system

In order to verify the correctness of the proposed model, this paper uses IEEE 39-bus system for analyzing the disturbance propagation. IEEE 39-bus system is well known as 10-machine New-England Power System. The IEEE 39-bus system has 10 generators, 12 transformers, 17 loads and 46 lines, as in [22]. The Gaussian white noise is added on the basis of the simulation data to simulate the small random perturbation due to the data acquisition and transmission.

IEEE 39-bus system shown in Fig. 4. We change the active power of 18^{th} node, reactive power is always 30Mar. According to the time series sampling, the active power change of 18^{th} node shown in Table 2, the other nodes load remains unchanged.

TABLE II. THE ACTIVE POWER PARAMETER OF 18^{TH} NODE

Time series sampling point	Active power of 18^{th} node（MW）
1-600	158
601-1200（Disturbance I）	208
1201-2000	158
2001-3000（Disturbance II）	658
3000-3500	158

Calculating the relative importance coefficient of each node after establishing the adjacency matrix. The relative importance coefficient of the 18^{th} node is 0.0229. From the 39 nodes of active power, reactive power, voltage, power angle to form a total of 137-dimensional samples. And use total 3500 time-series samples to build a random matrix for real-time analysis of the relationship between the various dimensions. $T_w = 300$. The linear eigenvalue statistics changes shown in Fig. 5.

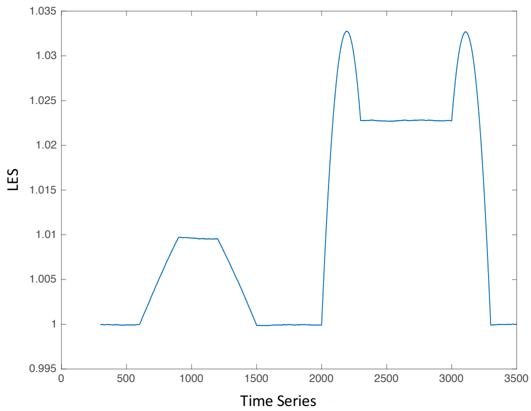

Fig. 5. Linear eigenvalue statistics

At 601^{st} sampling point, the linear eigenvalue statistic changes because of disturbance *I*. In the 2000^{th} sampling point is because the disturbance *II* occurs, then the system reaches a new stable state, and the correlation between the space-time data changed, the linear eigenvalue statistics also changed. Respectively, take disturbance *I*, disturbance *II* after the occurrence of the maximum value τ into the disturbance propagation model for analysis. After the disturbance *I* occurs, the established adjoint random matrix is used to obtain the correction coefficient. So the correction coefficient $K = 122.34$, the density of relative disturbed nodes is shown in Fig. 6.

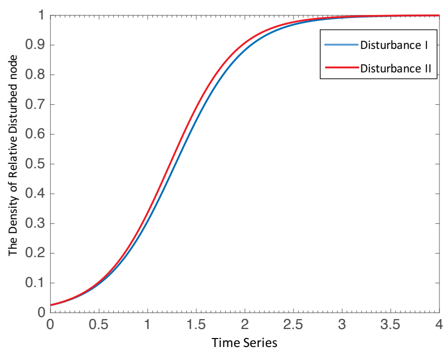

Fig. 6. The density of disturbed nodes in IEEE 39 system

Figure 6 starts from the disturbance occurred. It can be seen that the density of relative disturbed nodes of disturbance *II* is greater than that of disturbance *I* at the same time, and it can be concluded that the effect of disturbance *II* propagation is more serious than that of disturbance *I* in the system. In the simulation, the active power mutation of disturbance *II* is greater than that of disturbance *I* at the same node, so disturbance *II* is more critical than disturbance *I* to the system. Compared with the simulation result, the model can be used to evaluate the disturbance.

The random matrix is established with the voltage of each node and the voltage of all nodes. The visualization effect of the eigenvalue statistic mutation rate is shown in Fig.7.

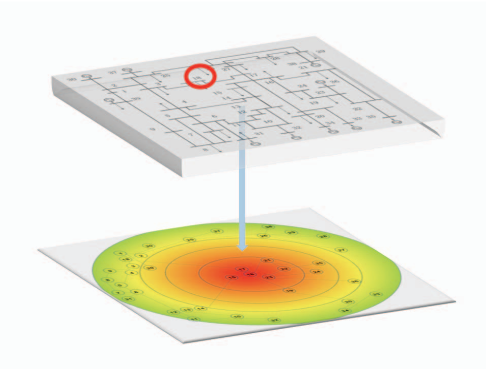

Fig. 7. Disturbance propagation visualization

VI. Conclusion

In this paper, a dynamic analysis of disturbance propagation is proposed, which aims to solve the problem of disturbance in power system. This method solves the dependence of mechanism model accuracy, and also solves the various simplifications and assumptions in the modeling process in traditional models which can't adequately reflect the actual operation in power system. The model we built not only considers the complex network topology of power system, but also analyzes the relationship between the spatial and temporal data. It transforms the data into knowledge, which helps to analyze and predict the disturbance propagation. The proposed propagation intensity can help early warning and thus take appropriate control measures.

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